

# HIGH REPETITION RATE CHARGING A MARX TYPE GENERATOR\*

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## Abstract

Resistive ladder networks are commonly used as the charging and isolation means for Marx type generators. The efficiency is limited to 50% and the charging time is long or equivalently the PRR (Pulse Repetition Rate) is low. The efficiency can be considerably improved by replacing the resistive ladder with inductor elements and the PRR is also improved. In this paper it is shown that by introducing mutual coupling,  $k$ , between the two parallel inductors in each stage of the ladder network, the effective inductance during the charging mode is decreased by a factor of  $(1-k)/(1+k)$ . Since it is feasible to achieve a coupling,  $k$ , on the order of 0.99, this speeds up the charging time by about an order of magnitude compared to uncoupled inductive charging. During the erected or discharge mode the inductors must provide isolation between stages and must not excessively rob energy from the energy store. The mutual coupling is beneficial in two ways. During the erected or discharge mode, it is shown that the effective inductance of the ladder elements are actually increased by a factor  $(1+k)$ . The Marx switches cause a re-arrangement of the coupled inductors from parallel during the charging to series during the discharge modes. This results in a much faster charging time, by reducing the effective inductance by  $(1-k)/(1+k)$ ; while providing an effective isolation inductance that is  $(1+k)$  greater than the uncoupled value. A practical design of the coupled inductor implementation and modeled simulations of the performance are compared to uncoupled and resistive charging.

## I. BACKGROUND INFORMATION

The Marx Type Generator has been used for many decades to generate very high voltages for single shot or low PRR applications. The basic principle is to charge a number of capacitors in parallel and then reconnect them in series by a set of switches; thereby boosting the voltage.

### A. Resistive Charging

The typical Marx configuration is shown in Figure 1. Resistors are used in a ladder network to provide for charging the capacitors in parallel. The resistance values must be high to prevent an objectionable loss of energy during the erected or discharge mode. The high value of resistance dictates a long charging period and thus a low PRR. In addition, resistive charging per se imposes a maximum efficiency of 50%. An empirical relation for the charging time to 99%,  $T(99\%)$ , normalized by the

time constant,  $(ts)$ , of a single stage, for an  $N$  stage Marx is:<sup>2</sup>

$$T(99\%)/ts = 0.0054N^3 + 1.87N^2 + 2.55N \quad (1)$$

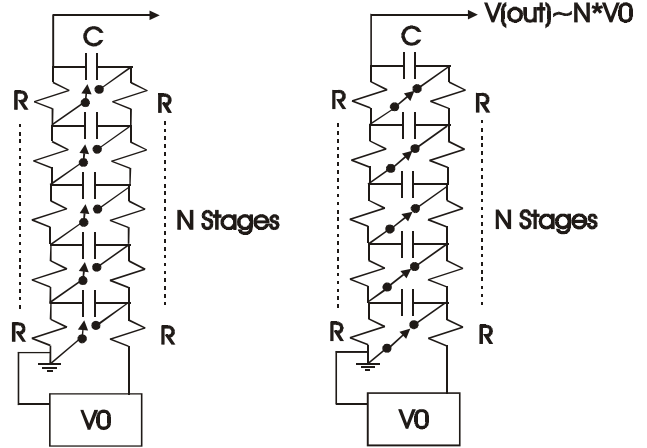


Figure 1. Traditional Marx Generator

### B. Simple Inductive Charging

If inductors,  $L$ , are used to replace the resistors,  $R$ , in the  $N$  stage Marx ladder circuit as shown in Figure 2, and a separate inductor,  $L_{chg}$ , is used to connect the ladder to the voltage source, both the efficiency and charging speed can be improved with respect to the resistively charged Marx.

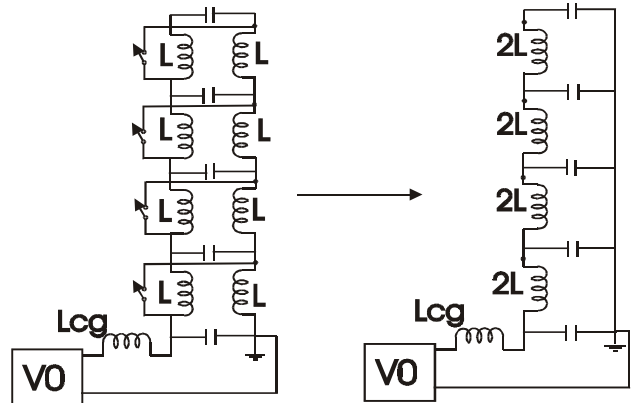


Figure 2. Marx With Resistors Replaced by Inductors

The Marx ladder is equivalent to a Guillemin type B PFN (Pulse Forming Network) or a lumped parameter transmission line. The electrical length of the equivalent transmission line is  $\tau_E = N(2LC)^{1/2}$ . There are two factors that must be considered in applying inductive charging.

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14. ABSTRACT <b>Resistive ladder networks are commonly used as the charging and isolation means for Marx type generators. The efficiency is limited to 50% and the charging time is long or equivalently the PRR (Pulse Repetition Rate) is low. The efficiency can be considerably improved by replacing the resistive ladder with inductor elements and the PRR is also improved. In this paper is it shown that by introducing mutual coupling, k, between the two parallel inductors in each stage of the ladder network, the effective inductance during the charging mode is decreased b1y a factor of (1-k)/(1+k). Since it is feasible to achieve a coupling, k, on the order of 0.99, this speeds up the charging time by about an order of magnitude compared to uncoupled inductive charging. During the erected or discharge mode the inductors must provide isolation between stages and must not excessively rob energy from the energy store. The mutual coupling is beneficial in two ways. During the erected or discharge mode, it is shown that the effective inductance of the ladder elements are actually increased by a factor (1+k). The Marx switches cause a re-arrangement of the coupled inductors from parallel during the charging to series during the discharge modes. This results in a much faster charging time, by reducing the effective inductance by (1- k)/(1+k); while providing an effective isolation inductance that is (1+k) greater than the uncoupled value. A practical design of the coupled inductor implementation and modeled simulations of the performance are compared to uncoupled and resistive charging.</b>		
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The rate at which the Marx ladder is charged results in an undesirable voltage ringing of the capacitors along the ladder; and when the Marx ladder is erected, the charging inductor,  $L_{chg}$ , robs energy from it. The capacitors will be charge to a peak value of twice the voltage of the d.c. power supply, i.e.  $2V_0$ . It is empirically determined that the peak to peak ripple normalized to  $V_p$  is a function of the ratio of the electrical length of the ladder,  $\tau_E$ , to the charging time,  $T_{CH}$ :

$$\frac{V_{pp}}{V_p} = 6.06 \left( \frac{\tau_E}{T_{chg}} \right)^{2.335} \quad (2)$$

The total equivalent inductance in parallel with the load,  $L_e$ , during the discharge mode is given by:

$$L_e = L \left( 1 + \frac{N-1}{2} \right) \quad (3)$$

The energy transferred to this inductance during the load period,  $\tau_L$ , is given by:

$$J_L = \frac{L_e}{2} I_\tau^2 = (1-\eta) 2NC(V_0)^2 \quad (4)$$

The value of the current,  $I_\tau$ , is calculated by:

$$I_\tau = \frac{2N(V_0)}{L_e} \int_0^\tau e^{-\frac{t}{\tau}} dt = \frac{2N(V_0)\tau}{L \left( 1 + \frac{N-1}{2} \right)} (1 - e^{-1}) \quad (5)$$

The term  $(1-\eta)$  is the fraction of the total energy that is transferred to the equivalent inductance during the load period. We can use equations 3,4, and 5 to determine the value of  $L$  consistent with the other parameters as:

$$L = \frac{N(1 - e^{-1})^2 \tau_L^2}{\left( 1 + \frac{N-1}{2} \right) (1-\eta) C} \quad (6)$$

Equation 2 can be used to determine the charging period,  $T_{chg}$ ; then since the Marx ladder is resonantly charged via the inductor  $L_{chg}$ , the value of  $L_{chg}$  is determined as:

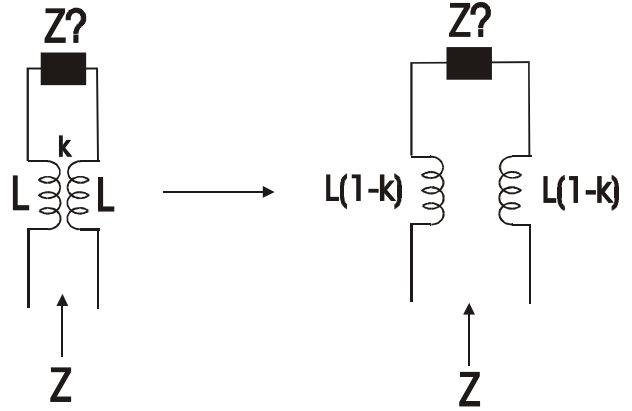
$$L_{chg} = \frac{T_{chg}^2}{\pi^2 NC} \quad (7)$$

## II. MARX WITH COUPLED INDUCTORS

The beneficial use of coupled inductors can be appreciated by examination of the behavior of a pair of equal coupled inductances when connected in parallel and in series, and by recognizing that the parallel to series transition will occur when a Marx ladder switches from the charge to discharge mode.

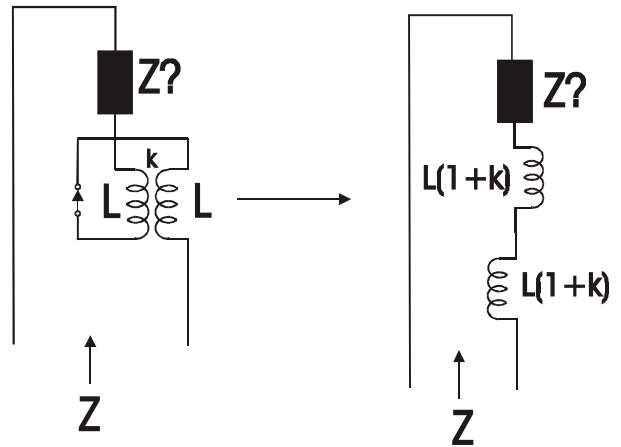
### A. Basic Coupled Inductor Theory

In Figure 3 a pair of equal inductors,  $L$ , with coupling coefficient  $k$ , is shown as they would be used in a stage of a Marx ladder in the charging mode. The right side diagram shows the equivalent circuit in terms of uncoupled equivalent inductors. The equivalent loop inductance is reduced to  $2L(1-k)$ . Considering that one should be able to accomplish a coupling coefficient on the order of 0.99, the effective length of the Marx ladder in the charging mode is reduced to about 10% of the uncoupled value.



**Figure 3.** Coupled Inductors in a Marx During the Charging Mode

The circuits in Figure 4 show the same inductors as they are arranged when the Marx is switched into the discharge mode.

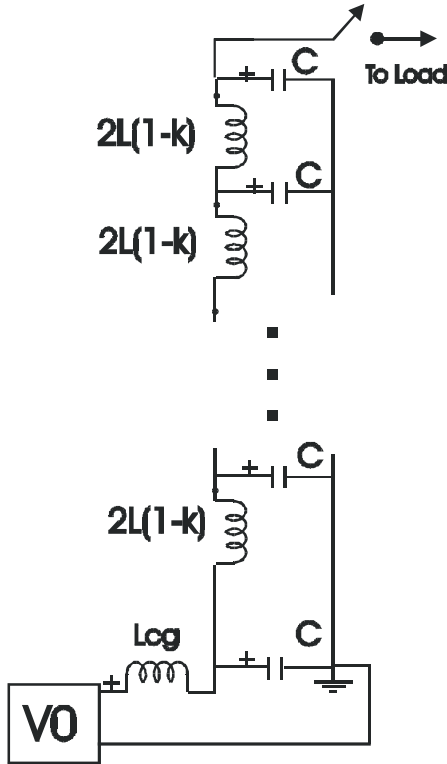


**Figure 4.** Coupled Inductors in a Marx During the Discharge Mode

Notice that the equivalent inductance bridging a stage in the discharge mode is  $2L(1+k)$ ; this is about twice as high as the inductance with no coupling.

### B. Coupled Inductors in a Marx Circuit

The equivalent circuit of a Marx in the charging mode with coupled inductors is shown in Figure 5. The effective inductance is reduced by the factor  $(1-k)$  and this results in the effective length of the Marx ladder,  $\tau_L$ , being shortened by a factor of  $(1-k)^{1/2}$ .



**Figure 5.** Equivalent Circuit of Marx With Coupled Inductors, in the Charging Mode

Notice that the inductance per section of the equivalent Type B PFN has been reduced by a factor of  $(1-k)$ , thereby reducing the electrical length,  $\tau_E$ , by a factor of  $\sim 10$  for  $k \sim .99$ . From equation (2) it is obvious that the charging time,  $T_{chg}$ , can now also be reduced by a factor of  $\sim 10$  and maintain the same performance on ringing. This is equivalent to a PRR increase of  $\sim 10$ .

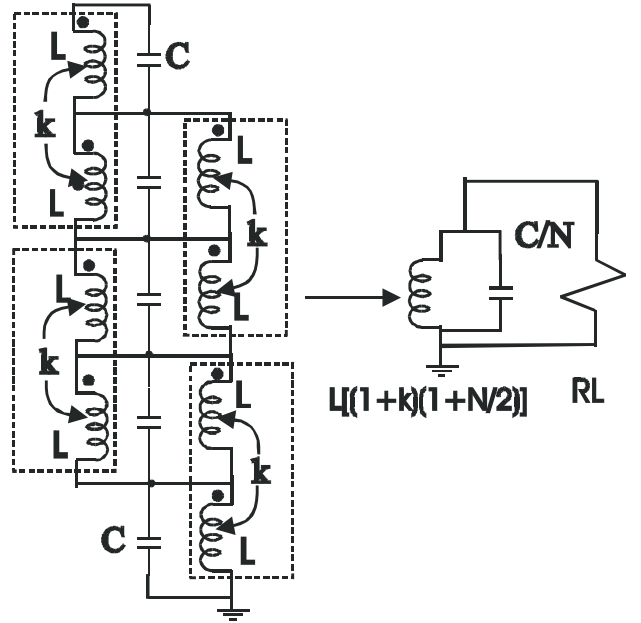
We now evaluate the total equivalent shunt inductance that appears in parallel with the load when the Marx is switched into the discharge mode. The equivalent circuit of the Marx in the discharge mode is shown in Figure 6.

The effective parallel inductance,  $L_{ek}$ , evaluated by circuit analysis is:

$$L_{ek} = Lk \left[ \left( 1 + k \right) \left( \frac{N}{2} + 1 \right) \right] \quad (8)$$

Compare this value of equivalent parallel inductance to the value in the case with no coupling in equation (3), we see that it is slightly more than a factor of two. Thus the energy robbed from the load is less and the value of  $Lk$  can be decreased. Therefore, by introducing coupling between the inductor pairs, we have achieved an increase in PRR of about an order of magnitude plus the effect of decreasing  $Lk$  slightly to maintain the same efficiency.

Circuit analysis simulations will be used to demonstrate this performance.



**Figure 6.** Equivalent Circuit of Marx With Coupled Inductors, in the Erected or Discharge Mode

### III. Circuit Analysis Simulation

The predicted performance can be verified by modeling examples with any circuit analysis and simulation program. The parameters of the example are:  $N=10$ ,  $\eta=0.95$ ,  $RL=50$ , and  $t_L=1\mu S$ .

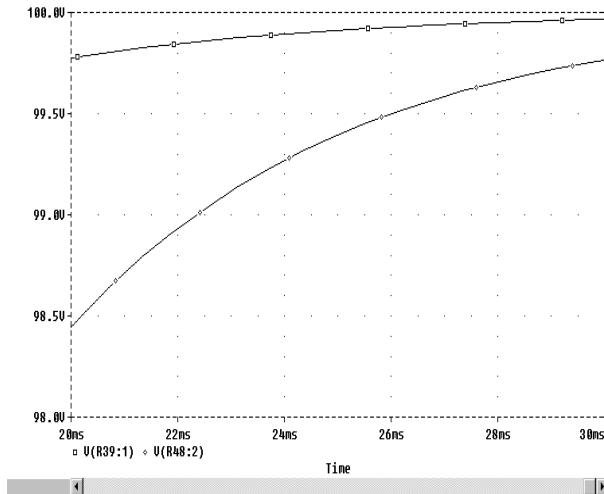
For the resistive charging case, equation (1), for a 99% charging voltage, determines the value of  $t_s$  as  $T_{chg}/195$ ,  $R=167$  and  $C=390nF$ . Thus,  $t_s=130\mu S$  and the charging time  $T_{chg}=25.4mS$ .

For the uncoupled inductor charging case, equation (2), for a peak to peak voltage ripple of 1%, has the value of  $T_{chg}/\tau_E=15.55$  and the value of  $L$  is determined by equation (6) as  $L=37.25\mu H$ . Thus, electrical length,  $t_E=53.87\mu S$  as determined by  $t_E=N(2LC)^{1/2}$ . Using equation (2) the ratio of  $T_{chg}/t_E=15.54$ , determines the charging time,  $T_{chg}=837.7\mu S$ . This charging time is much faster,  $\sim 30x$ , than the resistive case. In addition, the efficiency is nearly twice the 50% value of the resistive case.

For the coupled inductor charging case, assuming a value of  $k=0.99$ , and using equation (8) we determine that  $Lk=17.17\mu H$ . To determine the charging time we calculate  $t_{Ek} = N(2Lk(1-k)C)^{1/2}=3.66\mu S$ . The ratio of  $T_{chg}/t_{Ek}$  is the same as for the uncoupled case (15.55) therefore the  $T_{chg}$  for the coupled case is  $56.7\mu S$ . This is about 448 times faster than the resistive and 14.7 times the uncoupled cases.

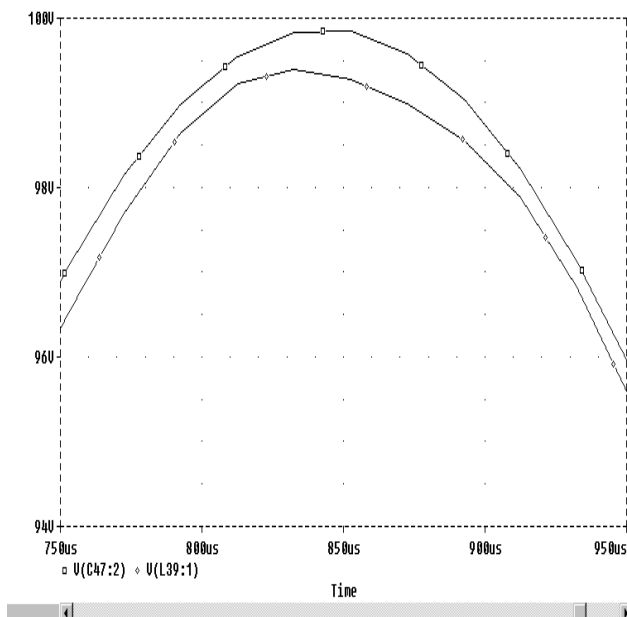
To check these results we use the circuit values for the above three cases and run a circuit analysis program to verify the predicted performance.

The charging voltage of the last capacitor in the 10 stage resistively configured Marx is shown in Figure 7. It shows that the voltage does reach the required level of 99% in 23mS close to the 25mS predicted.



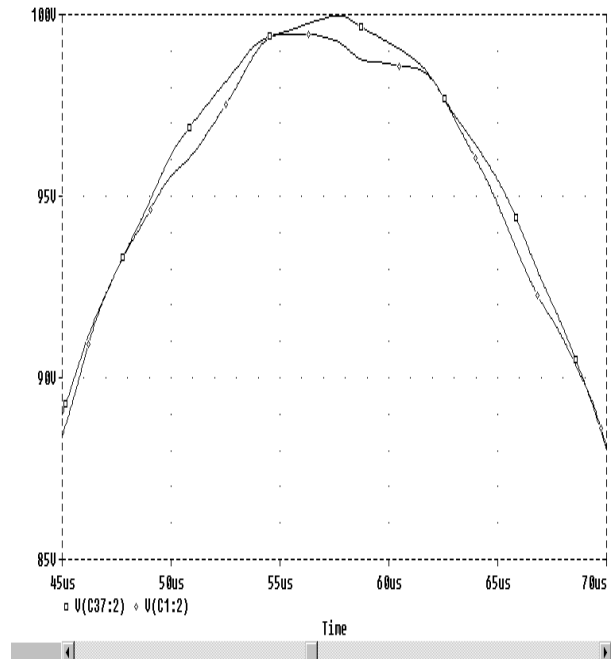
**Figure 7.** Charging Voltage of the First and Last Capacitors in a 10 Stage Resistive Marx Configuration

The charging voltage of the first and last capacitors in the 10 stage uncoupled inductor Marx configuration is shown in Figure 8. The traces are close to the design objective of  $\sim 1\%$ . The source voltage is set to 50% of the source voltage of the resistive case because of the gain of the resonant charging. The charging time to peak is, as predicted, about 840 $\mu$ S.



**Figure 8.** Voltage on First and Last Capacitors in a 10 Stage Marx Configured With Uncoupled Inductors

The charging voltage of the first and last capacitors in the 10 stage Marx configured with coupled inductors is shown in Figure 9. The coupling coefficient is taken to be  $k=0.99$ , and the other values as calculated above. The simulation agrees with the calculated charging time with in a reasonable margin, i.e.  $\sim 57\mu$ S.



**Figure 9.** Voltage on First and Last Capacitors in a 10 Stage Marx Configured With Coupled Inductors

## IV. SUMMARY

The analysis is verified by the circuit simulations and show that a resistive Marx configuration has a much lower PRR capability than an uncoupled inductor configuration. The uncoupled inductor configuration has an efficiency about twice as high as the 50% maximum efficiency of a resistive configuration. The coupled inductor configuration has both the high efficiency and a PRR capability that is about 448 times higher than the example 10 stage resistive and 14.7 time faster than the uncoupled inductive implementation. The relative advantage will vary somewhat with the number of stages but can be calculated using the methods explained.

## V. REFERENCES

- [1] J. Lehr and C. Baum, "Charging of Marx Generators", Circuit and Electromagnetic System Design Notes, Note 43, Air Force Research Laboratory, Directed Energy Directorate, 29 June 2000.
- [2] G. Glasoe and J. Lebacqz, "Pulse Generators", Volume 5, M.I.T. Radiation Laboratory Series, Boston Technical Publishers, 1964.